MAT 239 (Differential Equations)
Solutions to handout on Classification of Differential Equations

Consider the differential equation: $\frac{dQ}{dt} = -kQ$

Q	What is the dependent variable?
t	What is/are the independent variable(s)?
ODE	Is this an ODE or a PDE?
Yes	Is the DE linear?
1	What is the order of the DE?
No	Is $Q = 3e^{kt}$ a solution of the DE? If not, can you guess a solution?

$Q = 3e^{-kt}$ and $Q = e^{-kt}$ and $Q = Q_0 e^{-kt}$ for any Q_0 are all solutions.

Consider the differential equation: $(1 - x^2)y'' - 2xy' + 2y = 0$

y	What is the dependent variable?
x	What is/are the independent variable(s)?
ODE	Is this an ODE or a PDE?
Yes	Is the DE linear? *Note: DE is linear in the dependent variable, y.*
2	What is the order of the DE?
Yes	Is $y = x$ a solution of the DE? If not, can you guess a solution?

Consider the differential equation: $u_t + uu_x = 0$

u	What is the dependent variable?
t and x	What is/are the independent variable(s)?
PDE	Is this an ODE or a PDE? *Note: there is more than one independent variable.*
No	Is the DE linear? *Note: the uu_x term counts like u^2, making it nonlinear*
1	What is the order of the DE?
Yes	Is $u = 0$ a solution of the DE? If not, can you guess a solution?

Yes, $u = 0$ *is* a function, defined by $u(t, x) = 0$ for all t and all x is a function. It is a solution to the PDE since $0 = 0$ is true for all t and all x. Keep an eye out for these constant solutions to differential equations.

Consider the differential equation: $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin(\theta)$

θ	What is the dependent variable?
t	What is/are the independent variable(s)?
ODE	Is this an ODE or a PDE?
No	Is the DE linear? *Note: the $\sin(\theta)$ term is nonlinear in θ.*
2	What is the order of the DE?
No	Is $\theta = \frac{1}{2}gt^2$ a solution of the DE? If not, can you guess a solution?

The only solutions I can write down in closed form are $\theta = 0$ and $\theta = \pi$ (or any integer multiple of π). These are *constant* functions of t.