MAT 239 (Differential Equations), Prof. Swift
Worksheet 4 on Separation of Variables

1. Find the general solution to the ODE $y' = 2xy^2$.

This is not, technically, the *general* solution because the solution $y =$ __0__ is missing.

2. Using the general solution you just found, solve the IVP

$$y' = 2xy^2, \quad y(0) = 1.$$

After finding the particular solution, sketch the solution and fill in the blanks.
The interval of existence of this particular solution is __−1__ $< x <$ __1__.

1. $\frac{dy}{dx} = 2xy^2$

$\frac{dy}{y^2} = 2x\,dx$

$\int y^{-2}dy = \int 2x\,dx$

$\frac{y^{-1}}{-1} = x^2 + C$

$\frac{-1}{y} = x^2 + c$

$\boxed{y = \frac{-1}{x^2+C}}$

This does not contain the solution $y = 0$ for any c.

$\left(\lim_{x\to\infty} \frac{-1}{x^2+c} = 0\right)$

The graph of the solution to the IVP is

2. Plug in $x=0, y=1$

$1 = \frac{-1}{0^2+c} = \frac{-1}{c}$

$\therefore C = -1$

$\boxed{y = \frac{-1}{x^2-1}}$ OK

$\boxed{y = \frac{1}{1-x^2}}$ Better!

$1-x^2=0$ at $x = \pm 1$

$y = \frac{1}{1-x^2}$

NOT on graph of solution.

Default domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

~~The IVP is~~ the solution to the IVP is defined on the interval of existence $\boxed{-1<x<1}$ or $\boxed{(-1,1)}$