MAT 239 (Differential Equations), Prof. Swift
Worksheet 6 on Linear 1st Order ODEs

In this worksheet you will find the general solution to $xy' + 3y = 5x^2$.

$0+0 \neq 5x^2$ $\qquad$ $0+3k \neq 5x^2$

0. Is $y(x) = 0$ a solution? yes/no Is $y(x) = k$ a solution for any constant k? yes/no

1. Put the ODE $xy' + 3y = 5x^2$ into standard form and identify $p(x)$ and $g(x)$. What is the x value where p and/or g are not continuous? $\boxed{y' + \frac{3}{x}y = 5x}$, $p(x) = \frac{3}{x}$, $g(x) = 5x$

2. Compute *and simplify* the "magic" integrating factor $\mu(x)$.
see below.

3. Multiply both sides of the *standard form* of the ODE by $\mu(x)$.

p & g are continuous at all $x \neq 0$.

4. Compute $\frac{d}{dx}[\mu(x)y]$ leaving y as an unknown function of x. This should be the Left Hand Side (LHS) of that ODE you wrote in part 3. Magic! Rewrite the ODE as

$$\frac{d}{dx}[\mu(x)y] = \mu(x)g(x).$$

5. Integrate to find $\mu(x)y$ (don't forget the "+C") and then solve for y. This is the general solution!

2. $\mu(x) = \exp\left(\int \frac{3}{x}dx\right) = e^{3\ln|x| + c} = e^{c} \cdot e^{\ln|x|^3}$

$= \text{cnst} \cdot |x|^3$. we can choose cnst $= 1$ for $x > 0$ and cnst $= -1$ for $x < 0$

$\boxed{\mu(x) = x^3}$

3. $x^3\left(y' + \frac{3}{x}y\right) = x^3(5x)$

$\boxed{x^3y' + 3x^2y = 5x^4}$

4. $\frac{d}{dx}[x^3y] = \underbrace{3x^2y + x^3\frac{dy}{dx}}_{\text{LHS of equation 3!}}$, using the product rule.

original The ODE is equivalent to $\boxed{\frac{d}{dx}[x^3y] = 5x^4}$

5. $x^3y = \int 5x^4 dx = x^5 + C$. Solve for y:

$y = \frac{x^5 + C}{x^3}$, or $\boxed{y = x^2 + \frac{C}{x^3}}$