MAT 239 (Differential Equations), Prof. Swift
Worksheet 14, The Characteristic Equation

This whole worksheet is about the ODE $y'' + y' - 2y = 0$. Assume that the independent variable is x, so $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2y}{dx^2}$.

1. Write down one solution of the ODE.

2. Plug the function $y = e^{rx}$ into the ODE and find the equation that the constant r must satisfy so that $y = e^{rx}$ is a solution to the ODE. This is called the *characteristic equation* of the ODE, and it is super important.

3. Find the two roots of the characteristic equation. Call them r_1 and r_2.

4. Write down two different non-zero solutions of the ODE. Call them y_1 and y_2.

5. The general solution to the ODE is $y = c_1 y_1 + c_2 y_2$, where c_1 and c_2 are arbitrary constants. Write down the general solution using the solutions you found in part 4. Does this include the solution you guessed in question 1?