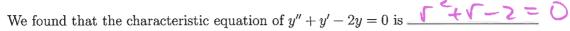
## MAT 239 (Differential Equations), Prof. Swift Worksheet 14, The Characteristic Equation

This whole worksheet is about the ODE y'' + y' - 2y = 0. Assume that the independent variable is x, so  $y' = \frac{dy}{dx}$  and  $y'' = \frac{d^2y}{dx^2}$ .

1. Write down one solution of the ODE. 9 = 0

2. Plug the function  $y = e^{rx}$  into the ODE and find the equation that the constant r must satisfy so that  $y = e^{rx}$  is a solution to the ODE. This is called the *characteristic equation* of the ODE, and it is super important.

$$y = e^{rx}$$
 $y'' = re^{rx}$ 
 $y'' = r^{2}e^{rx}$ 
 $e^{rx} + re^{rx} - 2e^{rx} = 0$ 
 $e^{rx} + re^{rx} - 2e^{rx} = 0$ 



3. Find the two roots of the characteristic equation. Call them  $r_1$  and  $r_2$ .

4. Write down two different non-zero solutions of the ODE. Call them  $y_1$  and  $y_2$ . 4.=e1x=px, 4,=e2x

5. The general solution to the ODE is 
$$y = c_1y_1 + c_2y_2$$
, where  $c_1$  and  $c_2$  are arbitrary constants. Write down the general solution using the solutions you found in part 4. Does this include the

solution you guessed in question 1?

6. Verify that the general solution you wrote down is a solution to the ODE for all  $c_1$  and all  $c_2$ . (Evaluate y' and y'', plug these into the ODE, and gather terms.)

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$$y = (10^{x} + 60^{2x})$$
 $y' = (10^{x} + 30^{2x})$ 
 $y'' = (10^{x} + 30^{2x})$ 
 $y'' = (10^{x} + 40^{2x})$ 
 $y'$