MAT 239 (Differential Equations), Prof. Swift
Worksheet 18, The General Solution of a LHODECC.

1. Suppose the characteristic equation of a LHODECC for $y(x)$ factors over the real numbers as $(r^2+4)^2(r-5)^3 = 0$. $r^2+4=0 \Rightarrow r^2=-4 \Rightarrow r=\pm 2i$

(a) What are the roots of the characteristic equation? $\boxed{r = \pm 2i, \pm 2i, 5, 5, 5}$

(b) What is the general solution of the LHODECC?

$y = C_1\cos(2x) + C_2\sin(2x) + C_3 x\cos(2x) + C_4 x\sin(2x) + C_5 e^{5x} + C_6 x e^{5x} + C_7 x^2 e^{5x}$

2. Suppose one solution of a 5th order LHODECC is $y(x) = 3x^2 + 4e^x\sin(2x)$.

(a) What are the roots of the characteristic equation of the LHODECC? $0, 0, 0, 1 \pm 2i$

(b) What is the simplest general solution of a LHODECC that contains the function $y(x) = 3x^2 + 4e^x\sin(2x)$?

$y = C_1 + C_2 x + C_3 x^2 + C_4 e^x\cos(2x) + C_5 e^x\sin(2x).$

Rule 1 says that the first guess for the *form* of the particular solution to a LNODECC, $L[y] = g(x)$ is the simplest general solution of a LHODECC that contains the function $g(x)$.

3. Use Rule 1 and Problem 2 to find the *form* of the particular solution to the LNODECC $y'' + y' + y = 3x^2 + 4e^x\sin(2x)$. This will have 5 undetermined coefficients A, B, C, D, and E.

$y_p = A + Bx + Cx^2 + De^x\cos(2x) + Ee^x\sin(2x).$

4. (a) Use Rule 1 to find the *form* of the particular solution to the LNODECC $y'' + y' + y = 3e^{2x}$. This will have a single undetermined coefficient A.

(b) Find a particular solution to that same LNODECC, $y'' + y' + y = 3e^{2x}$.

$y_p = Ae^{2x}$

$y_p' = 2Ae^{2x}$

$y_p'' = 4Ae^{2x}$

$4Ae^{2x} + 2Ae^{2x} + Ae^{2x} = 3e^{2x}$

$(4A + 2A + A)e^{2x} = 3e^{2x}$

$7A = 3$

$A = \frac{3}{7}$

$\therefore \boxed{y_p = \frac{3}{7}e^{2x}}$