

MAT 239 (Differential Equations), Prof. Swift
Worksheet 21, An Undamped, Undriven Oscillator

1. Suppose a ball stretches an ideal spring 6 inches. The ball is placed on the spring, where it oscillates up and down. Find the ODE for $y(t)$, the extension of the spring in feet beyond its equilibrium, after t seconds. Ignore friction and use $g = 32$ feet per second per second.

Hint: Let m be the mass of the ball. The spring constant k depends on m , but the ODE does not.

2. Write down the general solution to the ODE. It's OK to write down the solution by inspection.

3. Find the solution $y(t)$ to the ODE for arbitrary initial conditions $y(0) = y_0$ and $y'(0) = v_0$.

4. Find the particular solution to the ODE with the initial conditions $y(0) = -1$, $y'(0) = 8$.

5. Write the solution to 4 in the $y(t) = R \cos(\omega_0 t - \delta)$ form. Draw a picture in the c_1 - c_2 plane!

6. Sketch the solution to the IVP in problem 4 over at least two periods of the oscillation.