MAT 239 (Differential Equations), Prof. Swift
Worksheet 24, Test Review

1. Write down the general solution to $(D-2)^3(D-4)y = 0$. The independent variable is x.

$r = 2, 2, 2, 4$

$$y = (c_1 + c_2 x + c_3 x^2)e^{2x} + c_4 e^{4x}$$

2. Write down the form of a particular solution to $(D-2)^3(D-4)y = 5e^{2x} + 7\sin(4x)$

Rule 1: $y_p = Ae^{2x} + B\cos(4x) + C\sin(4x)$

Rule 2 modifies the e^{2x} terms:

$$\boxed{\cancel{y_p = Axe^{2x} + B\cos(4x) + C\sin(4x)}}$$

oops

$$\boxed{y_p = Ax^3e^{2x} + B\cos(4x) + C\sin(4x)}$$

3. Suppose that
$y_1(t)$ solves the IVP $y'' + e^t y' + \sin(t)y = 0$, $y(0) = 1$, $y'(0) = 0$,

$y_2(t)$ solves the IVP $y'' + e^t y' + \sin(t)y = 0$, $y(0) = 0$, $y'(0) = 1$, and

$y_3(t)$ solves the IVP $y'' + e^t y' + \sin(t)y = t^2$, $y(0) = 0$, $y'(0) = 0$.

Write down the solution to the IVP $y'' + e^t y' + \sin(t)y = t^2$, $y(0) = \alpha$, $y'(0) = \beta$.

It is OK to write down the solution by inspection:

$$\boxed{y = \alpha y_1(t) + \beta y_2(t) + y_3(t)}$$

Proof: Look for a solution $y = C_1 y_1(t) + C_2 y_2(t) + C_3 y_3(t)$

$y(0) = \alpha$, so $y(0) = C_1 y_1(0) + C_2 y_2(0) + C_3 y_3(0) \overset{\text{def}}{=} \alpha$

$C_1 \cdot 1 + C_2 \cdot 0 + C_3 \cdot 0 = \alpha$ $\boxed{C_1 = \alpha}$

similarly $y'(0) = C_1 y_1'(0) + C_2 y_2'(0) + C_3 y_3'(0) \overset{\text{def}}{=} \beta$, so $\boxed{C_2 = \beta}$

Finally, $L[y] = C_1 L[y_1] + C_2 L[y_2] + C_3 L[y_3] \overset{\text{def}}{=} g(t)$

$C_1 \cdot 0 + C_2 \cdot 0 + C_3 g(t) = g(t)$ $\boxed{C_3 = 1}$