MAT 239 (Differential Equations), Prof. Swift
Worksheet 25.5, More Power Series Review

The Taylor series for the function f, centered at a, is $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, provided you can differentiate f as often as you want. Recall that $f^{(n)}(a)$ is the nth derivative of f, evaluated at a, and $n!$ is "n factorial" defined by $0! = 1$ and $n! = n \cdot (n-1)!$ for positive integers.

1. Write out the first 3 terms ($n = 0$ up to $n = 2$, then write $+\cdots$) of the Taylor series using the exact notation of that sum. That is, continue the equation I've started.

$$f(x) = \frac{f^{(0)}(a)}{0!}(x-a)^0 +$$

2. Now, simplify that expression as much as possible. (Use the usual "prime" notation for derivatives, and evaluate the factorials.)

$f(x) =$

3. Use the formula from problem 2 to find the first three nonzero terms of the Taylor Series of $f(x) = \frac{1}{x} = x^{-1}$, centered at $a = 100 = 10^2$. (This has "$+\ldots$".)

4. Write down the degree 2 Taylor polynomial for that same function f, centered at $a = 100$.

5. Use the result of Problem 4 to get an approximation to $\frac{1}{99}$. Compare to the exact answer of $\frac{1}{99} = 0.01010101\ldots$.