Worksheet 25.5 More Power Series Review

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

1. write out first 3 terms.

$$f(x) = \frac{f^{(0)}(a)}{0!}(x-a)^0 + \frac{f^{(1)}(a)}{1!}(x-a)^1 + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \dots$$

2. Simplify: $0! = 1,\ 1! = 1,\ 2! = 2,$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots$$

3.

n	$f^{(n)}(x)$	$f^{(n)}(100)$	$C_n = \frac{f^{(n)}(100)}{n!}$
0	$f(x) = x^{-1}$	$100^{-1} = 0.01$	0.01
1	$f'(x) = -x^{-2}$	$-(100)^{-2} = \frac{-1}{10^4} = -0.0001$	-0.0001
2	$f''(x) = 2x^{-3}$	$2(100)^{-3} = \frac{2}{10^6} = .000002$	0.000001

So $f(x) = 0.01 - 0.0001(x-100) + 0.000001(x-100)^2 + \dots$

or $f(x) = \frac{1}{10^2} - \frac{1}{10^4}(x-100) + \frac{1}{10^6}(x-100)^2 + \dots$

4. $T_2(x) = 0.01 - 0.0001(x-100) + 0.00001(x-100)^2$.
(Just drop the "$+\dots$")

5. $\frac{1}{99} = f(99) \approx T_2(99) = 0.01 - 0.0001(99-100) + 0.000001(99-100)^2$

$= 0.01 + 0.0001 + 0.000001$

$\boxed{\frac{1}{99} \approx 0.010101}$ Pretty close to $\frac{1}{99} = 0.01010101\dots$