MAT 239 (Differential Equations), Prof. Swift
Worksheet 27, Systems of First Order ODEs

1. Let $A = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$. Is AB defined? Is BA defined? Compute the matrix product that is defined.

A is 2×1, B is 3×2 AB is not defined.

2×1 3×2

BA is defined, and it is a 3×1 matrix

3×2 2×1

$$BA = \begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0+5 \\ 8+0 \\ 4+15 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 19 \end{bmatrix}$$

2. Write $y'' + \frac{1}{Q}y' + y = \cos(\omega t)$ as a system of 2 first order ODEs for the position y and velocity v.

$y' = v$

$y'' = v'$

solve for y''

$y'' = -\frac{1}{Q}y' - y + \cos(\omega t)$

$v' = -\frac{1}{Q}v - y + \cos(\omega t)$

$$\boxed{\begin{aligned} y' &= v \\ v' &= -\tfrac{1}{Q}v - y + \cos(\omega t) \end{aligned}}$$

3. Write the system you found in problem 2 as a single matrix ODE $\frac{d}{dt}\mathbf{x} = A\mathbf{x} + \mathbf{g}(t)$. Start by defining the vector $\mathbf{x} = \begin{bmatrix} y \\ v \end{bmatrix}$.

$\vec{x} = \begin{bmatrix} y \\ v \end{bmatrix}$, so $\frac{d\vec{x}}{dt} = \frac{d}{dt}\vec{x} = \begin{bmatrix} y' \\ v' \end{bmatrix} = \begin{bmatrix} v \\ -\frac{1}{Q}v - y + \cos(\omega t) \end{bmatrix}$

$$\frac{d}{dt}\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} v \\ -y - \frac{1}{Q}v \end{bmatrix} + \begin{bmatrix} 0 \\ \cos(\omega t) \end{bmatrix}$$

Note that $v = 0 \cdot y + 1 \cdot v$

$$\boxed{\frac{d}{dt}\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -\frac{1}{Q} \end{bmatrix}\begin{bmatrix} y \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \cos(\omega t) \end{bmatrix}}$$