MAT 239 (Differential Equations), Prof. Swift
Worksheet 28, Eigenvalues and Eigenvectors

The eigenvalues/eigenvectors of a matrix A satisfy $A\mathbf{v} = \lambda\mathbf{v}$, $\mathbf{v} \neq \mathbf{0}$, and $\det(A - \lambda I) = 0$.

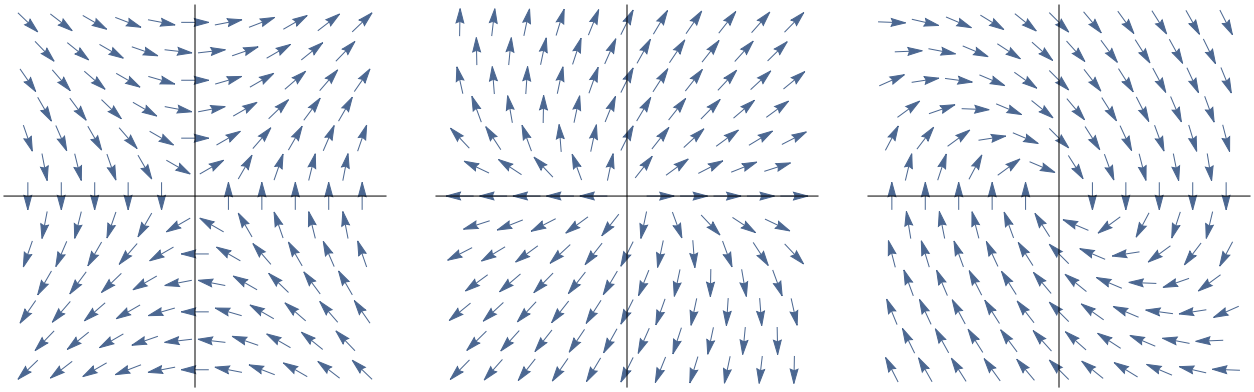
1. Let $A = \begin{bmatrix} 4 & 3 & 1 \\ 1 & 5 & 1 \\ 0 & 1 & 9 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Yes/No: Is $A\mathbf{v} = 3\mathbf{v}$?

Yes/No: Does that imply that 3 is an eigenvalue of A? Why not?

Yes/No: Is 3 an eigenvalue of A? (Hint: The determinant of $\begin{bmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 6 \end{bmatrix}$ is -6.) Why not?

2. The figure shows 3 vector fields $\mathbf{F}(\mathbf{x}) = A\mathbf{x}$. If A has real eigenvalues, draw the lines that are the eigenvector directions, and indicate if the corresponding eigenvalue is positive or negative.



2. Compute the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, and find the general solution to .

3. Solve the IVP $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, with the matrix A defined in problem 2.