MAT 239 (Differential Equations), Prof. Swift
The Swift Method for Repeated Eigenvalues, and Normal Modes

1. Use the Swift method to solve the IVP $\dfrac{d\mathbf{x}}{dt} = \begin{bmatrix} 7 & 1 \\ -4 & 3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$.

Note: Paul does this the hard way in example 1 of his notes on repeated eigenvalues.

2. Find the general solution to $\dfrac{d^2\mathbf{x}}{dt^2} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x}$.

Hint: Use the fact that if $A\mathbf{v} = -k^2\mathbf{v}$, then $\mathbf{x}(t) = (c_1 \cos(kt) + c_2 \sin(kt))\mathbf{v}$ is a solution to $\dfrac{d^2\mathbf{x}}{dt^2} = A\mathbf{x}$. This family of solutions is called a normal mode of the system.