4. Consider the ODE $\frac{dy}{dx} = x + y$. Open up the Slope Field App, found on our website. It shows the slope field of this 1st order ODE. Click around to show various solutions.

(a) Verify that $y = Ce^x - x - 1$ is a solution to the ODE for any constant C. This is the general solution since it can be shown that *every solution* to the ODE is obtained by some choice of C.

(b) Find the solution to the ODE with the initial conditions $y(0) = 1$, and find another solution with $y(0) = -1$.

(c) Go back to the Slope Field App. Click on "initial points", clear all curves, and plot the two solutions with these two initial conditions.

(d) Open Desmos Graphing Calculator, and plot the family of functions $y = Ce^x - x - 1$. Add a slider for the constant C and explore the solutions to $\frac{dy}{dx} = x + y$.

(a) $y = Ce^x - x - 1$

LHS: $\frac{dy}{dx} = Ce^x - 1$

RHS: $x + y = x + Ce^x - x - 1 = Ce^x - 1$

$\left.\right\}$ LHS $\overset{\checkmark}{=}$ RHS

(b) $y(x) = Ce^x - x - 1$

so $y(0) = Ce^0 - 0 - 1 \overset{\text{set}}{=} 1$

$C = 1$

so $\boxed{y(x) = e^x - x - 1}$ is the solution to $\frac{dy}{dx} = x + y$, $y(0) = 1$.

$y(0) = Ce^0 - 0 - 1 \overset{\text{set}}{=} -1$

$C - 1 = -1$

$\underline{C = 0}$

so $y = 0e^x - x - 1$, or $\boxed{y = x - 1}$ is the solution to $\frac{dy}{dx} = x + y$, $y(0) = -1$.