MAT 239 (Differential Equations), Prof. Swift
Worksheet on Differential Equations

1. Consider the ODE $\frac{dy}{dx} = 2y$, also written as $y' = 2y$.

(a) Verify that $y = 5e^{2x}$ is a solution to the ODE. $y' = 5e^{2x} \cdot 2 = 2(5e^{2x}) = 2y$ ✓

(b) Verify that $y = Ce^{2x}$ is a solution for every constant C. $y' = Ce^{2x} \cdot 2 = 2(Ce^{2x}) = 2y$ ✓

It is a fact that $y = Ce^{2x}$ is the general solution to the ODE. The general solution has 2 properties: (1) It is a solution for every choice of C. You already did that. (2) Every solution to the ODE is obtained by choosing C correctly.

(c) Find the particular solution to the Initial Value Problem $\frac{dy}{dx} = 2y$, $y(0) = 3$. (Use the general solution and find the C that works.) $3 = Ce^{2 \cdot 0} = C \therefore C = 3$, $\boxed{y = 3e^{2x}}$

2. Guess the general solution to the ODE $\frac{dy}{dx} = -y$. Verify property (1) for your guess. $y = Ce^{-x}$; $y' = Ce^{-x}(-1) = -(Ce^{-x}) = -y$ ✓

3. Solve the Initial Value Problem $\frac{dy}{dx} = -y$, $y(0) = 2$. $2 = Ce^{-0} = C \therefore C = 2$, $\boxed{y = 2e^{-x}}$