MAT 239 (Differential Equations), Prof. Swift
Worksheet 9 on 1st Order Modeling

Solve by inspection, after writing the ODE in the form $y' = k(k - A)$. Do not do any integrals. If you are tempted to do an integral, skip these problems are ask a neighbor how to solve these by inspection.

y

1. $\frac{dQ}{dt} = -\frac{1}{3}Q + 2$ $= -\frac{1}{3}(Q-6)$

(This might model the amount of salt in a tank of water.)

General solution to the ODE: $Q(t) = 6 + Ce^{-\frac{1}{3}t}$

Solution to IVP with $Q(0) = 2$: $Q(t) = 6 - 4e^{-\frac{1}{3}t}$

Solution to IVP with $Q(0) = 10$: $Q(t) = 6 + 4e^{-\frac{1}{3}t}$

2. $\frac{dP}{dt} = .2P - 1$ $= .2(P-5)$

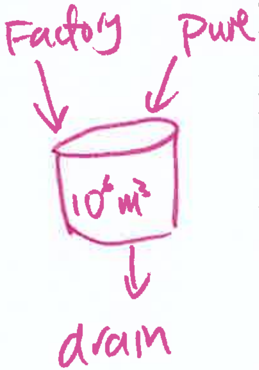
(This might model a population, before it runs out of food.)

General solution to the ODE: $P = 5 + Ce^{.2t}$

Solution to IVP with $Q(0) = 4$: $P = 5 - e^{0.2t}$

Solution to IVP with $Q(0) = 5$: $P = 5$

Solution to IVP with $Q(0) = 6$: $P = 5 + e^{+0.2t}$



3. In this problem you will model pollution of a lake. Assume that the volume of the lake is 10^6 cubic meters. Initially the water in the lake is pure, but at time $t = 0$ a factory starts polluting the lake by pumping in effluent with a concentration of 2 kilograms per cubic meter of "substance F". The effluent is pumped in at a rate of 100 cubic meters per day. There is also a source of pure water entering the lake at a rate of 900 cubic meters per day. The pollutant in the lake is well mixed, and the mixed water drains out of the lake at the rate of 1000 cubic meters per day. (Thus, the volume of the lake stays at a constant 10^6 cubic meters.)

$y(0)=0$

(a) Write down the IVP for $y(t)$, the kilograms of substance F in the lake after the factory has been open t days.

$$\frac{dy}{dt} = \text{rate in} - \text{rate out} \left(\frac{kg}{day}\right)$$

$$\text{rate in} = \frac{2\,kg}{m^3} \cdot 100 \frac{m^3}{day} = 2 \cdot 10^2 \frac{kg}{day}$$

$$\text{rate out} = \frac{y\,kg}{10^6\,m^3} \cdot 10^3 \frac{m^3}{day} = 10^{-3} y \frac{kg}{day}$$

IVP

$$\boxed{\frac{dy}{dt} = 2 \cdot 10^2 - 10^{-3} y = -10^{-3}(y - 2 \cdot 10^5), \quad y(0) = 0}$$

(b) Solve the IVP by inspection.

$$y(t) = 2 \cdot 10^5 + C e^{-10^{-3} t}$$

$y(0)=0$ gives $C = -2 \cdot 10^5$

$$\boxed{y(t) = 2 \cdot 10^5 - 2 \cdot 10^5 e^{-10^{-3} t}}$$

(c) How many kilograms of substance F are in the lake in the limit $t \to \infty$?

$$\boxed{y(t) \to 2 \cdot 10^5 \text{ as } t \to \infty.}$$

There are 20 thousand kg of substance F in the lake after a long time.