MAT 239, Differential Equations, Prof. Swift

Exam 1, on WeBWorK sets 1-8 will be Tuesday, February 17, 2026 (Week 6)

To study for the exam, start with this sample exam. I suggest you try the exam first before looking at the scanned solutions. Secondly, look at and understand the worksheets from class. Finally, go over the webwork problems. You can try the problems again and still get the green bar for a correct answer, even though the set is closed. If you don't have time to work all of the problems (haha) I suggest you try these:

Set 2: 8, 12 Set 3: 3, 11 Set 4: 1, 4 Set 5: 6 Set 6:1 Set 7: 3 Set 8: 3, 6

Here are the groundrules:

No notes are allowed. No calculators, phones, watches, or earbuds are allowed. Put your phone and watch in your backpack.

There are no hard integrals if you do the problems correctly. Show your work! If you don't show your work, or have errors in the work, I may take off points even if you have the correct answer.

All problems have equal weight.

The exam will begin and end on time. Many of you have classes in south campus before or after this one, so I will keep to the official schedule.

1. (a) Find the explicit solution to the initial value problem $\dfrac{dy}{dx} = xy^2, \quad y(0) = 1.$

(b) Sketch the solution and indicate the interval on which the solution is defined. (This is also called the interval of existence, or the interval of validity.)

2. (a) Find the solution to the IVP.

$$\frac{dy}{dt} + \frac{y}{t} = 4t^2, \quad y(1) = 0$$

(b) Sketch the solution and indicate the interval on which the solution is defined.

3. Find an implicit solution to the IVP

$$(\sin(y) + x^2)dx + (x\cos(y) + 1)dy = 0, \quad y(0) = 1$$

4. Suppose that you borrow \$10,000 for a car, at 5% per year interest. What should your monthly payment be so that you pay back the loan in 5 years?

Hint: Let $y(t)$ be the amount, in dollars, that you owe the bank after t years, and let P be your monthly payment. Assuming the interest is computed continuously, and the payments are made continuously, y satisfies the IVP.

$$\frac{dy}{dt} = \tfrac{1}{20}\, y - 12P, \qquad y(0) = 10000$$

Hint: $5\% = \frac{1}{20}$. Get an expression for P that you could evaluate with a calculator.

5. Put a "Y" or "N" in the blank, indicating if the ODE has the indicated property. Use the differential form to test exactness.

___ Separable ___ Linear ___ Exact $\quad \dfrac{dy}{dx} = \dfrac{y}{e^x}$ or $ydx - e^x dy = 0.$

___ Separable ___ Linear ___ Exact $\quad \dfrac{dy}{dx} = -y + x^2$ or $(y - x^2)dx + dy = 0.$

___ Separable ___ Linear ___ Exact $\quad \dfrac{dy}{dx} = \dfrac{x + e^y}{1 - xe^y}$ or $(x + e^y)dx + (xe^y - 1)dy = 0.$

6. Compute two steps of Euler's method for the IVP $\frac{dy}{dx} = 2xy$, $y(0) = 1$ with $h = \frac{1}{2}$.
That is, compute y_1 and y_2, that approximate $y(.5)$ and $y(1)$.

7. Find the equilibrium solutions, and indicate if they are stable, unstable, or semistable, for the ODE

$$\frac{dy}{dt} = (y + 1)^2(y - 1)$$

Sketch the phase portrait on the y axis, and sketch several solutions in the t-y plane.

8. Solve the initial value problem for $y(t)$ by inspection and indicate the solution by putting a letter in the blank. You do not have to show your work.

A. $y' = -2y, \quad y(0) = 1$
B. $y' = 2(y - 1), \quad y(0) = 1$
C. $y' = -y, \quad y(0) = 2$
D. $y' = -2y, \quad y(0) = 0$
E. $y' = -2y + 4, \quad y(0) = 0$
F. $y' = 2, \quad y(0) = 1$

____ $y = 0$
____ $y = 2e^{-t}$
____ $y = 2t + 1$
____ $y = e^{-2t}$
____ $y = 2 - 2e^{-2t}$
____ $y = 1$

9. Match the ODE to the slope field. Put a letter in the box. Also, sketch the solution for each ODE with the initial condition $y(0) = 0$.

A. $y' = 1 - x^2$ B. $y' = 1 - y^2$ C $y' = y - x$

D. $y' = x^2 - 1$ E. $y' = y^2 - 1$ F. $y' = x - y$

