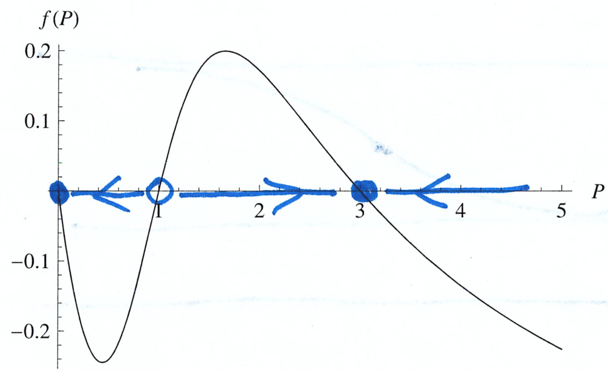
MAT 239 (Differential Equations), Prof. Swift
Worksheet 10 on Autonomous 1st Order ODEs

The population, $P(t)$, of green-eyed frogs on an island (measured in thousands of frogs) at time t (measured in months) obeys the differential equation

$$\frac{dP}{dt} = f(P),$$

where the graph of f is shown below.

$P=0$, $P=1$, $P=3$ } Equilibrium Solutions

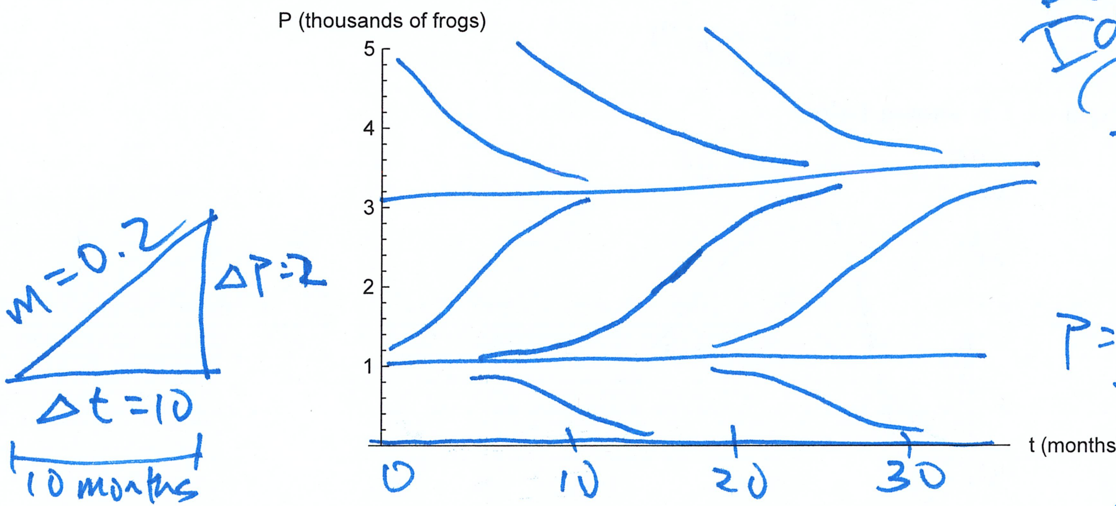


(a) Write down this ODE's equilibrium solutions (also called constant solutions).

(b) Draw the phase portrait for this ODE on the P axis, with a closed dot for a stable equilibrium, an open circle for an unstable equilibrium, and arrows indicating if P is increasing or decreasing between equilibria.

(c) Sketch the graph of several solutions of this ODE. Include the equilibrium solutions, and and a few non-constant solutions. After sketching the solutions, put a few numbered tick marks on the t axis to indicate the approximate time scale. (Hint: The maximum value of $\frac{dP}{dt}$ is 0.2 thousand frogs per month.)

Bonus answers for I did Not ask here.



P=3 is a stable equilibrium

P=1 is an unstable equilibrium

P=0 is a stable equilibrium

(d) Fill in the blanks to tell the story of this model.

If the initial population of green-eyed frogs, $P(0)$, is below 1 thousand, then the frogs will go extinct. On the other hand, if the initial population is above 1 thousand frogs, then the frog population will eventually reach approximately 3 thousand.