# MAT 239 (Differential Equations), Prof. Swift
# Worksheet 22, Damped Oscillators

Using the dimensionless time $\bar{t} = \omega_0 t$, the ODE for $y(\bar{t})$ of a damped harmonic oscillator is

$$y'' + \tfrac{1}{Q}\, y' + y = 0$$

where the *quality factor* $Q$ is a positive dimensionless constant.

1. Write down the characteristic equation for $r$, and solve it using the quadratic formula. Simplify.

2. What value of $Q$ is critical damping? (The characteristic equation has repeated roots.)

3. Recall that $\sqrt{1+x} = (1+x)^{1/2} \approx 1 + x/2$ for $x$ near zero. Use this to find an approximation the roots of the characteristic equation when $Q$ is large.


# MAT 239 (Differential Equations), Prof. Swift
# Worksheet 22, Damped Oscillators

Using the dimensionless time $\bar{t} = \omega_0 t$, the ODE for $y(\bar{t})$ of a damped harmonic oscillator is

$$y'' + \tfrac{1}{Q}\, y' + y = 0$$

where the *quality factor* $Q$ is a positive dimensionless constant.

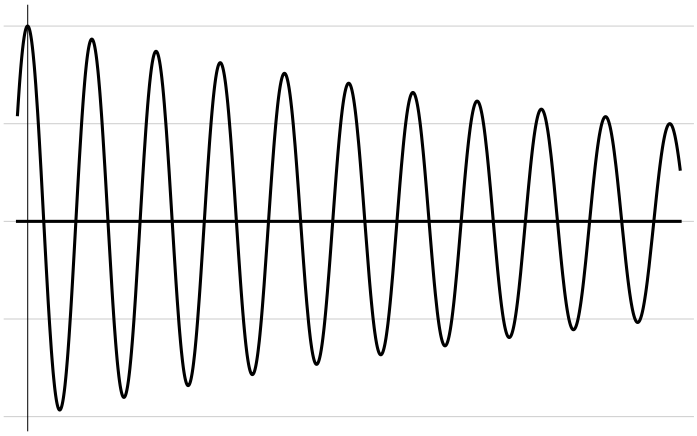
1. Write down the characteristic equation for $r$, and solve it using the quadratic formula. Simplify.

2. What value of $Q$ is critical damping? (The characteristic equation has repeated roots.)

3. Recall that $\sqrt{1+x} = (1+x)^{1/2} \approx 1 + x/2$ for $x$ near zero. Use this to find an approximation the roots of the characteristic equation when $Q$ is large.

4. For very large $Q$ (see the NIST web site) we can approximate $\sqrt{1+x} \approx 1$ in the formula for the roots of the characteristic equation. Write down the approximate general solution $y(\bar{t})$ when $Q$ is very large, using the $R$-$\delta$ form.

5. Use your answer to part 4 to estimate $Q$ for the oscillator with $y$ as a function of time plotted below. Note that no time scale is given, and no $y$ scale is given. The time scale might be nanoseconds or millennia, but you *can* figure out $\bar{t}$. The $y$ scale might be nanometers or light years, but it doesn't matter. Get an expression for $Q$ and then evaluate it with a calculator.



4. For very large $Q$ (see the NIST web site) we can approximate $\sqrt{1+x} \approx 1$ in the formula for the roots of the characteristic equation. Write down the approximate general solution $y(\bar{t})$ when $Q$ is very large, using the $R$-$\delta$ form.

5. Use your answer to part 4 to estimate $Q$ for the oscillator with $y$ as a function of time plotted below. Note that no time scale is given, and no $y$ scale is given. The time scale might be nanoseconds or millennia, but you *can* figure out $\bar{t}$. The $y$ scale might be nanometers or light years, but it doesn't matter. Get an expression for $Q$ and then evaluate it with a calculator.

