

MAT 362, Set 4, Sp 2012

§ 3.1 #1a. $x_0=0, x_1=0.6, x_2=0.9$

$f(x) = \cos(x)$. Find an interpolating Poly of degree 1 & 2.
use it to approximate $f(0.45)$ & find the absolute error.

$$L_{1,0} = \frac{x-0.6}{0-0.6}, \quad L_{1,1} = \frac{x-0}{0.6-0} \quad \text{use } x_0 \text{ and } x_1 \text{ only.}$$

$$P_1(x) = f(0)L_{1,0} + f(0.6)L_{1,1} = \cos(0) \left(\frac{x-0.6}{-0.6} \right) + \cos(0.6) \frac{x}{0.6}$$

$$P_1(x) = \frac{x}{-0.6} + 1 + \frac{\cos(0.6)}{0.6} x = \left[\frac{\cos(0.6)-1}{0.6} \right] x + 1 = -0.2911x + 1$$

$$P_1(0.45) = 0.869002... \quad \text{Abs. error } |f(0.45) - P_1(0.45)| \approx 0.0314$$

Sorry, the answer in the back of the book is wrong!
Now, $n=2$. (This can be checked with a calculator!)

$$L_{2,0} = \frac{(x-0.6)(x-0.9)}{(0-0.6)(0-0.9)} \quad L_{2,1} = \frac{(x-0)(x-0.9)}{(0.6-0)(0.6-0.9)} \quad L_{2,2} = \frac{(x-0)(x-0.6)}{(0.9-0)(0.9-0.6)}$$

$$P_2(x) = \overset{1}{\cos(0)} \cdot \frac{(x-0.6)(x-0.9)}{(-0.6)(-0.9)} + \cos(0.6) \frac{x(x-0.9)}{(0.6)(0.6-0.9)} + \cos(0.9) \frac{x(x-0.6)}{(0.9)(0.9-0.6)}$$

or good enough.

$$P_2(x) = -0.431x^2 - 0.0325x + 1 \quad \leftarrow \text{again - book is wrong!}$$

$$P_2(0.45) = 0.8981... \quad \text{Abs. error} = |\cos(0.45) - P_2(0.45)| \approx 0.00235$$

#2b. Similar to 1, with $x_0=1, x_1=1.25, x_2=1.6$

$f(x) = \sqrt[3]{x-1} = (x-1)^{1/3}$. Approximate $f(1.4)$ using $P_1(x)$ & $P_2(x)$.

use $x_1=1.25$ and $x_2=1.6$ for $P_1(x)$, since 1.4 is between x_1 and x_2 .

$$P_1(x) = f(1.25) \frac{(x-1.6)}{(1.25-1.6)} + f(1.6) \frac{(x-1.25)}{(1.6-1.25)} = 0.60992x - 0.13244$$