Project 2, MAT 362

Prof. Swift

Please do the project with groups of 2 or 3 people. Ask my permission (via email) if you want to work alone or with a group of 4.

The project is a paper. It can be a Mathematica notebook, with lots of text cells, or a .pdf file along with the Mathematica .nb file or Matlab .m file that you use.

In the course of the paper, do the things listed in A - D below. Remember, though, that the paper should stand alone, and it should make sense to someone who has not read this assignment.

Your report should have a title, alphabetical list of authors, a short introduction of the topic and contents of the report, the numerical results, and a discussion of the results. Reports should be professional in appearance and format, but need not be exceedingly long. The emphasis should be on developing and testing simple, correct code, and presenting the results clearly.

Submit the project as email attachments. I will send an email message to the class with a subject like "Project 2 for MAT 362", and one person from the group should reply, with a cc to the other members of the group.

A. Write a computer function that can approximate $\int_a^b f(x)\,dx$ using each of the three methods: Composite Trapezoid Rule, Composite Midpoint Rule, and Composite Simpson's Rule. The arguments of the function should be f, n, a, b (in the notation of Section 4.4), and some argument to indicate the method. You may modify the Mathematica Notebook numInt.nb available on our website, or write your own MATLAB program. Test your programs with problems 1f, 3f, and 5f in Section 4.4. (The answers are in the back of the book.) The code should have enough comments to be understandable to me.

B. Evaluate $\displaystyle\int_1^3 \frac{x}{x^2+4}\,dx$ exactly, by finding and antiderivative. Use this exact result when computing the relative error of your estimates. For each of three methods, find the smallest value of n so that the relative error of the estimate is less than 0.1%. Do this by trial and error, or by looping on n. To make the comparison fair, also report the number of function evaluations implied by that n. Let m be the number of function evaluations, and note that $m = n + 1$ for the Composite Trapezoid and Simpson's Rules, and $m = n/2 + 1$ for the Composite Midpoint Rule.

C. Consider the integral $\displaystyle\int_1^2 \frac{1}{x}\,dx$. Use the theorems from Section 4.4 to get upper and lower bounds for the signed error (estimate - actual value) of these three methods, as a function of n. Note that this is a pencil-and paper calculation, and h should be eliminated in favor of n.

D. Now, approximate $\displaystyle\int_1^2 \frac{1}{x}\,dx$ and verify that the error bounds you found in part C hold. For each of the three methods, make a table of m (the number of function evaluations), n, the estimate of the integral, the lower bound of the signed error from part C, the signed error, and the upper bound of the signed error from part C. In your tables, let m be all the allowed values for that method with $m \leq 9$.