MAT 461 (Partial Differential Equations)
Fall 2019, with Prof. Swift

First Assignment v.3, due Friday, Aug. 30

1. This problem concerns a nonhomogeneous second order linear ODE $\mathcal{L}[y] = g(t)$. Suppose that:
$y_1(t)$ satisfies the ODE with the initial conditions $y(0) = 1$, $y'(0) = 0$,
$y_2(t)$ satisfies the ODE with the initial conditions $y(0) = 0$, $y'(0) = 1$, and
$y_3(t)$ satisfies the ODE with the initial conditions $y(0) = 0$, $y'(0) = 0$.
Find the solution to the ODE with the initial conditions $y(0) = \alpha$, $y'(0) = \beta$. Justify your answer.
Note: Sometimes y_1 and y_2 denote solutions to the associated homogeneous ODE $\mathcal{L}[y] = 0$. In this problem they satisfy the original nonhomogeneous ODE.

2. Solve the ODE. Give the general solution if no initial condition is given, otherwise give the unique solution to the initial value problem (IVP). You might be able to solve (a)-(g) by inspection. Do *not* attempt to find the general solution to part (g).

(a) $\dfrac{dy}{dt} = 3y$, $y(0) = 2$.

(b) $\dfrac{d^2y}{dt^2} = -9y$

(c) $\dfrac{dy}{dt} = 2(y-1)$

(d) $\dfrac{d^3y}{dt^3} = 0$

(e) $\dfrac{d^2y}{dt^2} = 4y$, $y(0) = 3$, $y'(0) = 10$.

(f) $\dfrac{d^2y}{dt^2} = -4y$, $y(0) = 3$, $y'(0) = 10$.

(g) $\dfrac{d^2y}{dt^2} + t^2\dfrac{dy}{dt} + y^2 = 0$, $y(0) = 0$, $y'(0) = 0$.

(h) $\dfrac{dy}{dt} + \dfrac{3}{t}y = t$, $y(1) = 0$.

3. (a) Solve the IVP $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 2y = 0$, $y(0) = 1$, $y'(0) = 0$.

(b) Solve the IVP $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 2y = 0$, $y(0) = 0$, $y'(0) = 1$.

(c) Solve the IVP $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 2y = 0$, $y(0) = \alpha$, $y'(0) = \beta$ for arbitrary α and β.

4. Solve the IVP

$$\frac{d^2y}{dt^2} = \lambda y, \quad y(0) = 0, \; y'(0) = 1.$$

Consider the three cases $\lambda > 0$, $\lambda = 0$, and $\lambda < 0$ separately.

5. Find the values of λ for which this *boundary value problem* (BVP) has a nonzero solution.

$$\frac{d^2y}{dx^2} = \lambda y, \quad y(0) = y(\pi) = 0.$$

Consider the three cases $\lambda > 0$, $\lambda = 0$, and $\lambda < 0$ separately.
Note: An IVP always* has a unique solution. Not so for a BVP.

* You may want to search for Picard's Existence Theorem for a careful statement.

6. Suppose that a coffee cup is placed on a counter at $t = 0$. The coffee is originally at 200 degrees, and the room is at 70 degrees. After 10 minutes ($t = 10$) the coffee is at 185 degrees. Assuming that Newton's law of cooling holds, find the temperature of the coffee at a function of t, and determine when the temperature of the coffee will be 100 degrees.