MAT 461, Partial Differential Equations
Prof. Swift

Here is a summary of formulas from MAT 239 (Differential Equations). This is a very condensed summary that I gave as a formula sheet for the final exam. It leaves out a lot of details. There is a link to Paul's notes on our web site.

First order ODEs

These formulas concern solutions to $y' = f(y, t)$.

A separable ODE can be written $f(y)dy = g(t)dt$. Integrate both sides (adding a constant immediately) and try to solve for y to get an explicit solution $y = \varphi(t)$. An implicit solutions has the form $F(y, t) = c$.

The standard form for a first order linear ODE is $y' + p(t)y = g(t)$. This can be solved using the integrating factor $\mu(t) = \exp(\int p(t)\, dt)$.

There is a technique for solving Exact ODEs, and lots of other techniques.

Linear ODEs of order 2 or higher

These formulas concern solutions to $p_n(t)y^{(n)} + \cdots + p_1(t)y' + p_0(t)y = g(t)$.

The general solution of an nth order linear homogeneous ODE is $y = c_1y_1(t) + \cdots + c_ny_n(t)$, where $\{y_i(t) \mid 1 \leq i \leq n\}$ is a linearly independent set of solutions.

If the ODE is homogeneous ($g = 0$) with constant coefficients, then a real root λ of the characteristic equation corresponds to a solution $y = e^{\lambda t}$ of the ODE.

A complex conjugate pair of roots $\lambda = a \pm ib$ of the characteristic equation corresponds to two solutions $y = e^{at}\cos(bt)$ and $y = e^{at}\sin(bt)$of the ODE.

Repeated roots introduce factors of t to get linearly independent solutions.

For non-homogeneous ODEs ($g(t)$ is not the zero function), then the general solution is $y = y_h + y_p$, where y_h is the generenral solution to the associated homogeneous equation, and y_p is a particular solution to the non-homogeneous ODE.