

$$\vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y}$$

so  $\vec{\nabla} u = \hat{i} \frac{\partial u}{\partial x} + \hat{j} \frac{\partial u}{\partial y}$

If  $u = u(r, \theta)$

$$\vec{\nabla} u = \hat{i} \frac{\partial}{\partial x} u(r, \theta) + \hat{j} \frac{\partial}{\partial y} u(r, \theta)$$

$$= \hat{i} \left( \frac{\partial u}{\partial r} \cdot \frac{dr}{dx} + \frac{\partial u}{\partial \theta} \cdot \frac{d\theta}{dx} \right) + \hat{j} \left( \frac{\partial u}{\partial r} \cdot \frac{dr}{dy} + \frac{\partial u}{\partial \theta} \cdot \frac{d\theta}{dy} \right)$$

$$= \hat{i} \left( \frac{\partial u}{\partial r} \cos \theta + \frac{\partial u}{\partial \theta} \left( -\frac{\sin \theta}{r} \right) \right) + \hat{j} \left( \frac{\partial u}{\partial r} \sin \theta + \frac{\partial u}{\partial \theta} \left( \frac{\cos \theta}{r} \right) \right)$$

$$\vec{\nabla} u = \left[ \hat{i} \left( \cos \theta \frac{\partial}{\partial r} - \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \right) + \hat{j} (\dots) \right] u$$

$$\vec{\nabla} = \hat{i} \left( \cos \theta \frac{\partial}{\partial r} - \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \right) + \hat{j} (\dots)$$

$$\vec{\nabla} = \left( \hat{i} \cos \theta + \hat{j} \sin \theta \right) \frac{\partial}{\partial r} + \left( \hat{i} \left( -\frac{\sin \theta}{r} \right) + \hat{j} \left( \frac{\cos \theta}{r} \right) \right) \frac{\partial}{\partial \theta}$$

