$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y}$$

so $\vec{\nabla} u = \hat{\imath}\frac{\partial u}{\partial x} + \hat{\jmath}\frac{\partial u}{\partial y}$

If $u = u(r,\theta)$

$$\vec{\nabla} u = \hat{\imath}\frac{\partial}{\partial x}u(r,\theta) + \hat{\jmath}\frac{\partial}{\partial y}u(r,\theta)$$

$$= \hat{\imath}\left(\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta}\cdot\frac{\partial \theta}{\partial x}\right) + \hat{\jmath}\left(\quad\right)$$

$$= \hat{\imath}\left(\frac{\partial u}{\partial r}\cos\theta + \frac{\partial u}{\partial \theta}\left(-\frac{\sin\theta}{r}\right)\right) + \hat{\jmath}\left(\quad\right)$$

$$\vec{\nabla} u = \left[\hat{\imath}\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right) + \hat{\jmath}(\ldots)\right] u$$

$$\vec{\nabla} = \hat{\imath}\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right) + \hat{\jmath}(\ldots)$$

$$\vec{\nabla} = \left(\hat{\imath}\cos\theta + \hat{\jmath}\ldots\right)\frac{\partial}{\partial r} + \left(\hat{\imath}\left(-\frac{\sin\theta}{r}\right) + \hat{\jmath}(\ldots)\right)\frac{\partial}{\partial \theta}$$