COMPLEX EIGENVALUES

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Example: Find the general solution and sketch the phase portrait for

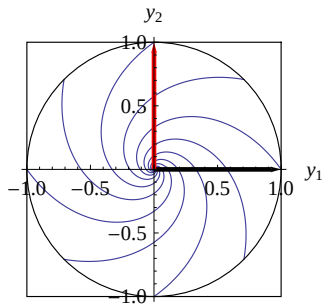
$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 3 \\ -1 & 0 \end{bmatrix} \mathbf{x}$$

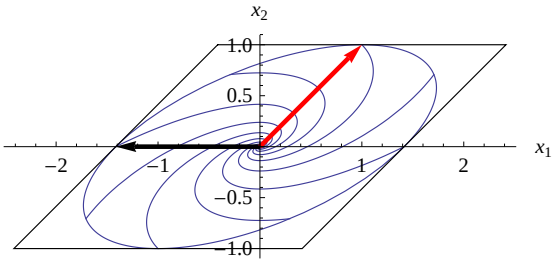
In class, we found that an eigenvalue of A is $\lambda = -1 + \sqrt{2}i$, and an associated eigenvector is $\mathbf{w} = [1 - \sqrt{2}i, 1]^T = [1, 1]^T + i[-\sqrt{2}, 0]^T$.

Therefore the general solution is

$$\mathbf{x} = r_0 e^{-t} \left\{ \begin{bmatrix} -\sqrt{2} \\ 0 \end{bmatrix} \cos(\sqrt{2}t + \theta_0) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(\sqrt{2}t + \theta_0) \right\}$$

The phase portrait in the $\mathbf{y}$ and $\mathbf{x}$ coordinates are shown in this figure





Note that the solutions always rotate counter-clockwise in the $\mathbf{y}$ coordinates, and in the $\mathbf{x}$ coordinates they go from $\mathbf{v}$ (black) toward $\mathbf{u}$ (red).

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