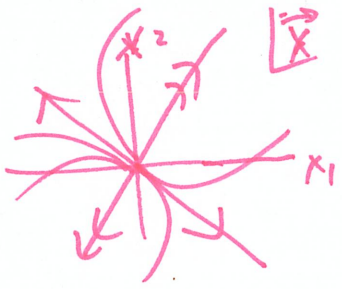
Set 3, Perko's ODE book.

1.2 1ac, 2, 3ac, 5, 7

1a. $A=\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$ $A\begin{bmatrix}1\\1\end{bmatrix}=4\begin{bmatrix}1\\1\end{bmatrix}$, $A\begin{bmatrix}1\\-1\end{bmatrix}=2\begin{bmatrix}1\\-1\end{bmatrix}$

$\vec{x}=c_1e^{4t}\begin{bmatrix}1\\1\end{bmatrix}+c_2e^{2t}\begin{bmatrix}1\\-1\end{bmatrix}$ $\dot{\vec{y}}=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}\vec{y}$







$B=P^{-1}AP=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$ for $P=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

1c. $A=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ $A\begin{bmatrix}1\\1\end{bmatrix}=0\begin{bmatrix}1\\1\end{bmatrix}$, $A\begin{bmatrix}1\\-1\end{bmatrix}=-2\begin{bmatrix}1\\-1\end{bmatrix}$

$\vec{x}(t)=c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2e^{-2t}\begin{bmatrix}1\\-1\end{bmatrix}$

$P=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ $P^{-1}AP=B=\begin{bmatrix}0 & 0\\ 0 & -2\end{bmatrix}$

$\dot{y}_1=0$

$\dot{y}_2=-2y_2$





2. $\lambda = 1, 2, -1$ for $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

$A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = -1\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

evec for $\lambda = 1$ is harder

$A - 1I = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix} \rightsquigarrow$ [illegible]

$R_3 \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $R_2 \to R_2 - R_3$

evec

$\vec{v} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ gives $P^{-1}AP = B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\vec{x}(t) = c_1 e^{t}\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + c_2 e^{2t}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 e^{-t}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

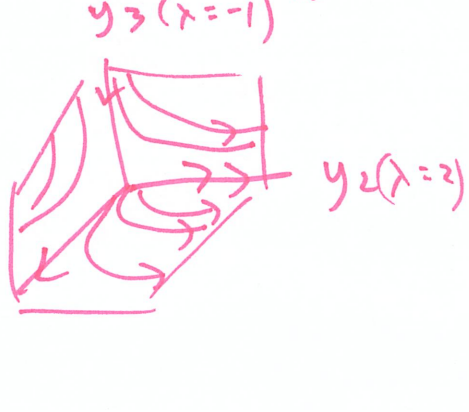
$E^s = \text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$

$E^u = \text{span}\left\{\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 \;\middle|\; x_1 = x_2 = z \right\}$ $2x_1 + x_3 = 0$



too hard to draw.

In y-coordinates



3c. $\vec{x}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix} \vec{x}$ (A)

$\dddot{x} = -2x + \dot{x} + 2\ddot{x}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -2x_1 + x_2 + 2x_3$

$A\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $A\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = -\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\vec{x} = c_1 e^{2t}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_3 e^{t}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

or $P^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 1 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$

4. (a) $P\vec{c} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ or $P\vec{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Prob 2: solve $\begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\vec{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Solve for $\vec{c}$ $\vec{x} = \frac{3}{2}e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{2}e^{2t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\vec{c} = \begin{bmatrix} \frac{1}{2} \\ 3 \\ 2.5 \end{bmatrix}$

5. all eigenvalues negative $\vec{x} = \frac{1}{2}e^{t}\begin{bmatrix} -\frac{3}{2} \\ 1 \end{bmatrix} + 3e^{2t}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 2.5\,e^{-t}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

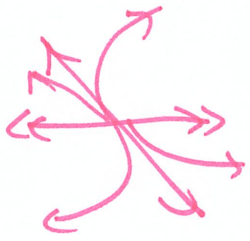
7.



(a)



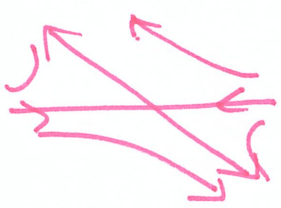
(b)



(c)



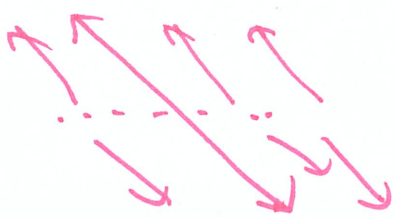
(d)



(e)



f



3a. $\ddot{x} + \dot{x} - 2x = 0$

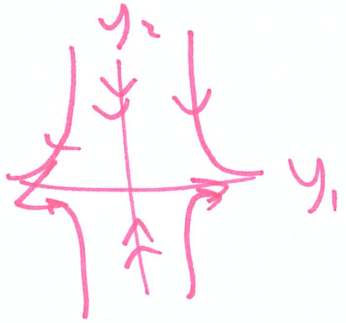
$\dot{x}_1 + \dot{x}_1 - 2x_1 = 0 \qquad \dot{x}_1 = x_2$

$\dot{x}_1 = -\dot{x}_1 + 2x_1$ so $\dot{x}_2 = -x_2 + 2x_1$

$$\dot{\vec{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ +2 & -1 \end{bmatrix}}_{A} \vec{x} \quad \text{or} \quad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2x_1 + x_2 \end{aligned}$$

$$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \lambda = 1, \qquad A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\vec{x} = C_1 e^{t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$





extra:

$\vec{x}' = A\vec{x}$ has general sol'n

$$\vec{x} = c_1 e^{-t}\begin{bmatrix}1\\0\end{bmatrix} + c_2 e^{-2t}\begin{bmatrix}1\\1\end{bmatrix}$$

$$P = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix},\quad B = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$$

Satisfies

$$P^{-1}AP = B$$

$$A = PBP^{-1} = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}$$

$$\boxed{A = \begin{bmatrix}-1 & -1\\ 0 & -2\end{bmatrix}}$$