Complex eigenvalues.

$a, b \in \mathbb{R}$.

If $A \in \mathbb{R}^{n\times n}$ has an eigenvalue $\lambda = a+ib$ $(b \neq 0)$

then it also has an eigenvalue $\overline{\lambda} = a-ib$.

We can consider $A \in \mathbb{C}^{n\times n}$. This is called "Complexification", summarized as "$\mathbb{R}$ is a complex number".

We can find $P \in \mathbb{C}^{n\times n}$ so that

$P^{-1}AP = B$ is in Jordan canonical

Form. If $J_r(a+ib)$ is a block in B, then

$J_r(a-ib)$ is also a block in B.

ex. $n=2$. $B = \begin{bmatrix} a+ib & 0 \\ 0 & a-ib \end{bmatrix}$, $n=4$ $B = \begin{bmatrix} J_2(a+ib) & 0 \\ 0 & J_2(a-ib) \end{bmatrix} = \begin{bmatrix} a+ib & 1 & 0 & \\ 0 & a+ib & & \\ & & a-ib & 1 \\ 0 & & 0 & a-ib \end{bmatrix}$