example: Suppose $A \in \mathbb{R}^{4\times 4}$ has eigenvalues $1\pm 2i$, $1\pm 2i$. There are 2 possibilities for $B$:

$$\begin{bmatrix} 1+2i & & & \\ & 1+2i & & \\ & & 1-2i & \\ & & & 1-2i \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1+2i & 1 & & \\ 0 & 1+2i & & \\ & & 1-2i & 1 \\ & & 0 & 1-2i \end{bmatrix}.$$

$P = [\vec{w}_1\ \vec{w}_2\ \vec{w}_3\ \vec{w}_4]$

$\vec{w}_j \in \mathbb{C}^4$

$\vec{w}_3 = \overline{\vec{w}_1}$

$\vec{w}_4 = \overline{\vec{w}_2}$

Choose this to save work.

$s_1 = 2$ for $\lambda = 1+2i$

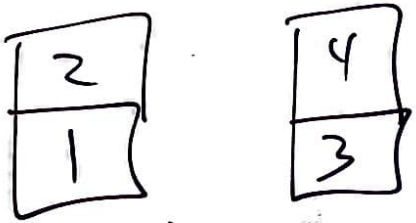



$\lambda = 1+2i$ $\quad \lambda = 1-2i$

$(A - (1+2i)I_2)\vec{w}_1 = \vec{0}$

$(A - (1+2i)I_2)\vec{w}_2 = \vec{0}$

Choose $\vec{w}_1, \vec{w}_2$ so $\{\vec{w}_1, \vec{w}_2\}$ are L.I over $\mathbb{C}$.




$s_1 = 1$

$s_2 = 2$ for $\lambda = 1+2i$

$\lambda = 1+2i$ $\quad \lambda = 1-2i$

$(A - (1+2i)I)\vec{w}_2 = \vec{w}_1$

$(A - (1+2i)I)\vec{w}_1 = \vec{0}$

$\vec{w}_1 \neq \vec{0}$