MAT 665, Differential Equations, Prof. Swift
Exam 1, 10-12-2009, on Sections 1.1-1.8

This test is open book and open notes. A graphing calculator is allowed, but you shouldn't need it. You need not deal with irrational numbers, nor do you need to invert matrices, in this exam. Ask me if you have any questions!

Do your work on your own paper. Write your name on each sheet, and indicate clearly what problem you are doing. You may do more than one problem on a sheet, but please use only the front of each page.

In problems 1 and 2, find the general solution to $\dot{\mathbf{x}} = A\mathbf{x}$ and sketch the phase portrait fairly accurately, where A is the given matrix:

$$\text{Problem 1:} \quad A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \qquad \text{Problem 2:} \quad A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

Problems 3 and 4 both concern the matrix

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 4 & 0 & 0 \\ -1 & 1 & 2 \end{bmatrix}$$

which has the eigenvalue 2 with multiplicity 3. You may use the facts that

$$(A - 2I) = \begin{bmatrix} 2 & -1 & 0 \\ 4 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad (A - 2I)^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & -1 & 0 \end{bmatrix}$$

and $(A - 2I)^3 = 0$.

3. Find the solution to the initial value problem $\dot{\mathbf{x}} = A\mathbf{x}$, $\mathbf{x}(0) = \mathbf{x}_0$, using the "$S + N$" method. Note: If you find yourself *computing* the inverse of some matrix, you are working too hard.

4. Find the general solution to $\dot{\mathbf{x}} = A\mathbf{x}$ for the same matrix A, using the Jordan canonical form method.

5. Suppose that A is a 3×3 real matrix, and you know the reduced row-echelon form of $A - 2I$ and of $A - (3 + i)I$ are as follows:

$$\text{rref}(A - 2I) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{rref}(A - (3 + i)I) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -i \\ 0 & 0 & 0 \end{bmatrix}$$

Find the general solution of $\dot{\mathbf{x}} = A\mathbf{x}$.