MAT 665, Differential Equations, Prof. Swift
Exam 1, 2026-03-06, on Sections 1.1-1.6

Name: _____

No book, no notes. Electronic devices, including cell phones and graphing calculators, are not allowed.

You may bring one page of notes, with writing on both sides.

Unless I warn you otherwise, you need not deal with fractions or irrational numbers.

You do you need to invert matrices in this exam. In fact, computing the inverse of any matrix might result in a small reduction of points, since it indicates that you are not a true disciple of mine.

Ask me if you have any questions!

1. Find the general solution to $\dot{\mathbf{x}} = A\mathbf{x}$, and sketch the phase portrait fairly accurately, where A is the given matrix:

$$A = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}$$

2. Assume that A is a 2×2 real matrix with eigenvalues $\pm i$. Furthermore, an eigenvector for i is $\begin{bmatrix} 1+i \\ 1 \end{bmatrix}$.

Write down the general solution to $\dot{\mathbf{x}} = A\mathbf{x}$ using the "r_0, θ_0" form, and sketch the phase portrait in the $\mathbf{x}$ plane fairly accurately.

3. Let A be the 3×3 matrix

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) Find the eigenvalues and eigenvectors of A.
(b) Find the general solution to $\dot{\mathbf{x}} = A\mathbf{x}$.
(c) Find a matrix P, and a diagonal matrix B, such that $P^{-1}AP = B$.
(d) Sketch the 3D phase portrait in $\mathbf{y}$ space, where $\mathbf{x} = P\mathbf{y}$.

In case you need more room, I have repeated the matrix from problem 3 here, and given you more room to do the computations.

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. The solution to this problem *does* involve fractions and irrational numbers! Consider the ODE

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix} \mathbf{x}$$

(a) Use the "Swift method" to solve the ODE with the initial condition $\mathbf{x}(0) = (1, -1)$.

(b) Use the "newly discovered Swift method" to evaluate $\exp(At)$. As a hint, recall that the ODE with the arbitrary initial condition $\mathbf{x}(0) = \mathbf{x}_0$ is $\mathbf{x}(t) = \exp(At)\mathbf{x}_0$.