§1.6 Prob. 3 in Perko.

(a) List the 5 upper JCFs for a 4×4 matrix with eigenvalue λ of multiplicity 4 (Note λ must be real. Assuming A is real.)

(b) what is the form of the solution to $\dot{\vec{x}} = A\vec{x}$.

$4=4$

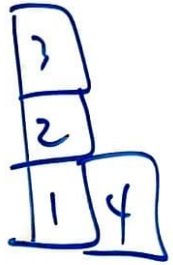
$$B=\begin{bmatrix}\lambda & 1 & 0 & 0\\ 0 & \lambda & 1 & 0\\ 0 & 0 & \lambda & 1\\ 0 & 0 & 0 & \lambda\end{bmatrix}$$



$\delta_1=1, \delta_2=2, \delta_3=3, \delta_4=4$

$4=3+1$

$$B=\begin{bmatrix}\lambda & 1 & 0 & 0\\ 0 & \lambda & 1 & 0\\ 0 & 0 & \lambda & 0\\ 0 & 0 & 0 & \lambda\end{bmatrix}$$



$\delta_1=2, \delta_2=3, \delta_3=4 \ (\because \delta_4=4)$

Note:

$$e^{Bt}=e^{\lambda t}\begin{bmatrix}1 & t & \frac{t^2}{2} & 0\\ 0 & 1 & t & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$\vec{y}=e^{Bt}\vec{c}$ is general

$4=2+2$



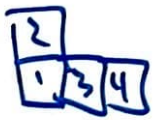
$$B=\begin{bmatrix}\lambda & 1 & & \\ 0 & \lambda & & \\ & & \lambda & 1\\ & & 0 & \lambda\end{bmatrix}$$

$\delta_1=2$

$\delta_2=4$

$\therefore (\delta_3=\delta_4=4)$

$4=2+1+1$



$$B=\begin{bmatrix}\lambda & 1 & & \\ 0 & \lambda & & \\ & & \lambda & \\ & & & \lambda\end{bmatrix}$$

$\delta_1=3$

$\delta_2=4$

$(\because \delta_3=\delta_4=4)$

$4=1+1+1+1$

$B=\lambda I$

$\delta_1=1, \delta_2=2, \delta_3=3, \delta_4=4$



example of part b.

for 

use $e^{\beta t}$ shown above.

$$\vec{x}=e^{\lambda t}\left(\vec{v}_1(c_1+c_2t+c_3\tfrac{t^2}{2}) + \vec{v}_2(c_2+c_3t) + \vec{v}_3 c_3 + \vec{v}_4 c_4\right)$$