MAT 667 (Dynamical Systems)
Homework # 3 due Monday, Feb. 20, 2017, in class.

This covers sections 1.5 - 1.8

Read the May article.

In the the logistic family of maps, find a numerical approximation of the value of a that has a superstable period 8 orbit. (You can do this with the iteratedMap1.nb *Mathematica* notebook on our web site.) I will tell you that $3.55 < a < 3.56$. Experiment with the values of a to refine the estimate to one more decumal place, as in $3.55* < a < 3.55*$.

Do problems 1.11, 1.12, and 1.14 on p. 37.

Do exercise T1.12 on pg. 27.

In problems 1-3, let $f : [0, 1) \to [0, 1)$ be defined by $f(x) = 2x \pmod 1 = 2x - \lfloor 2x \rfloor$.

1. Use the map f to calculate the first 10 bits of the binary expansion of $\sqrt{2}/2$ using a calculator or a computer. Describe the method you use.

2. Write the binary number $0.0010101\ldots_2 = 0.0\overline{01}_2$ as a fraction of two binary integers, and as fraction of two base 10 integers. Answer: $\frac{1}{110_2} = \frac{1}{6}$.

3. (a) Find all the period 3 points of the map f. Give them in the binary expansion, and as a ratio of base 10 integers.
(b) Write down the binary expansion of a point x_0 that looks like it is period 2 for several iterations, but which is eventually periodic with period 3.

In problems 4-6, let t to denote the tent map, defined as

$$t : [0, 1] \to [0, 1];\ t(x) = \begin{cases} 2x & \text{if } 0 \le x \le \frac{1}{2} \\ 2 - 2x & \text{if } \frac{1}{2} \le x \le 1 \end{cases}$$

4. Let $x \in [0, 1]$ can be written in base 2 as $x = 0.b_0b_1b_2b_3\ldots_2$, possibly ending in all 1's, as in $1 = 0.\overline{1}_2$. Find the base 2 expansion of $t(x)$. You may use the "bit flip" notation $b_i^* := 1 - b_i$. Show that $t(\frac{1}{2}) = 1$ using either of the representations $\frac{1}{2} = 0.1\overline{0}_2$ and $\frac{1}{2} = 0.0\overline{1}_2$.

5. Find the base 2 expansions of the periodic points with period less than or equal to 3, using the results of problem 4.

6. Sketch t, $t^2 = t \circ t$, and t^3. This will look a lot like Figure 1.10 in the book (p. 22). Get an expression for t^2 as a piecewise defined function, and determine the exact values of the period 2 points, as fractions. Show that the base 2 representation of the period 2 points you found in problem 5 agree with what you just found.