MAT 667 (Dynamical Systems)

Homework #4, due Monday, March 27, 2017

Exercise T2.9 on pg. 81, Exercise T2.10 on pg. 89, and Exercises 2.1, 2.3, 2.5, 2.7, 2.8, 2.9 on p. 98.

1. Find an expression for Fibonacci Numbers F_n defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$. Do this by re-writing the problem as $\mathbf{v}_{n+1} = A\mathbf{v}_n$, with $\mathbf{v}_n = (F_n, F_{n+1})$, and using the diagonalization of A to compute A^n for arbitrary n.

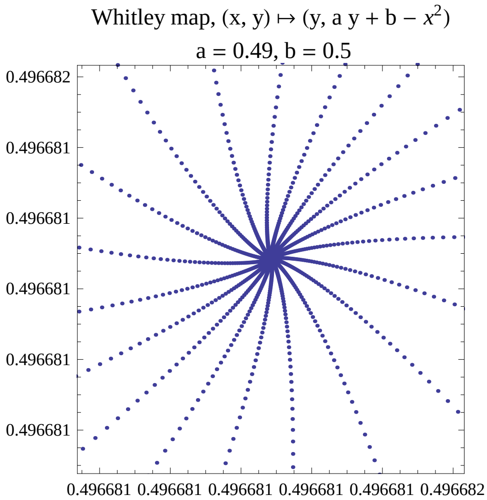
In the rest of this homework, the Whitley map $w_{a,b} : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$w_{a,b}(x, y) = (y, ay + b - x^2).$$

2. Find the fixed points of the Whitley map as a function of the parameters a and b. Let $(x_\pm, y_\pm)$ denote the fixed points, which depend on a and b. Sketch the region in the a-b plane where fixed points exist.

3. (a) Compute $Dw_{a,b}(x, y)$, and evaluate the Jacobian Matrix at the fixed point. Denote this as $A_\pm := Dw_{a,b}(x_\pm, y_\pm)$.

(b) This figure shows a trajectory with $a = 0.49$ and $b = 0.5$ values shown.



Numerically evaluate the two fixed points at these parameters, and verify that the fixed point shown is (x_+, y_+). Find the linearization A_+, and compute the eigenvalues of A_+, at these parameters. Explain the trajectory shown.

(c) Find the line segment in the a-b plane where the eigenvalues of A_+ are on the unit circle in the complex plane. Indicate the points where the eigenvalues are $\{1, 1\}$, $\pm i$, $\pm e^{i2\pi/3}$, and $\{-1, -1\}$.

(d) Make an accurate drawing of the a-b plane, showing the curves and points you have found in problems 2 and 3(c).

4. Play with the iterated2DMap.nb program on the web site from Feb. 20. Reply to my email with a gif of an interesting attractor not in the Feb. 22 Whitley Map page.