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On Finite Limit Sets for Transformations on the Unit Interval

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An infinite sequence of finite or denumerable limit sets is found for a class of many-to-one transformations of the unit interval into itself. Examples of four different types are studied in some detail; tables of numerical results are included. The limit sets are characterized by certain patterns; an algorithm for their generation is described and established. The structure and order of occurrence of these patterns is universal for the class.

i	k_i	P_i	$Q_\lambda(x)$
1	2	R	3.2360680
2	4	RLR	3.4985617
3	6	RLR^3	3.6275575
4	7	RLR^4	3.7017692
5	5	RLR^2	3.7389149
6	7	RLR^2LR	3.7742142
7	3	RL	3.8318741
8	6	RL^2RL	3.8445688
9	7	RL^2RLR	3.8860459
10	5	RL^2R	3.9057065
11	7	RL^2R^3	3.9221934
12	6	RL^2R^2	3.9375364
13	7	RL^2R^2L	3.9510322
14	4	RL^2	3.9602701
15	7	RL^3RL	3.9689769
16	6	RL^3R	3.9777664
17	7	RL^3R^2	3.9847476
18	5	RL^3	3.9902670
19	7	RL^4R	3.9945378
20	6	RL^4	3.9975831
21	7	RL^5	3.9993971