$$(f^k)'(x_0) = (f \circ f^{k-1})'(x_0)$$

$$= f'(f^{k-1}(x_0)) \cdot (f^{k-1})'(x_0)$$

$$= f'(x_{k-1}) \cdot (f^{k-1})'(x_0)$$

$$= f'(x_{k-1}) \cdot (f \circ f^{k-2})'(x_0)$$

$$= f'(x_{k-1}) \cdot f'(f^{k-2}(x_0)) \cdot (f^{k-2})'(x_0)$$

$$= f'(x_{k-1}) \cdot f'(x_{k-2}) \cdot (f \circ f^{k-3})'(x_0)$$

$$\vdots$$

$$= f'(x_{k-1}) \cdot f'(x_{k-2}) \cdot \ldots \cdot f'(x_0)$$

Result.

for $x_\ell = f^\ell(x_0)$

$$\boxed{(f^k)'(x_0) = \prod_{\ell=0}^{k-1} f'(x_\ell)}$$

this is true even if $f^k(x_0) \neq x_0$.