Northern Arizona University
College of the Environment, Forestry, and Natural Sciences
Department of Mathematics and Statistics

MAT 667 (Dynamical Systems) Syllabus for Spring 2023

Class 9455, MWF 10:20-11:10 in AMB 146

Instructor Information

Instructor: Jim Swift Jim.Swift@NAU.edu AMB 110 523-6878

Office Hours: Tu 11:30-12:30, W 11:30-1:00, Th 2:45-3:45, F 11:30-1:00. If these times are inconvenient, you can make an appointment, or drop by my office any time. E-mail is always a good way to contact me.

Websites: Our class website is oak.ucc.nau.edu/jws8/classes/667.2023.1/. I use the class web site for most electronic communication. I use BbLearn for grades and for posting documents, like practice exams and scanned solutions, that I don't want the world to see.

Course Description

Textbook: There is no required textbook. We will read a few journal articles, including the seminal paper of Edward Lorenz about what we now call the Lorenz Equations. Some recommended resources are

- Chaos: An introduction to dynamical systems, by Alligood, Sauer, and Yorke. This is available as an eBook through Cline Library.
- Nonlinear Dynamics and Chaos, by Steven Strogatz.

Prerequisite: MAT 239 and MAT 431, or consent of instructor.

Content and Course Objectives: The course is about iterated maps and, to a lesser extent, ordinary differential equations. These model time evolution, hence the name "dynamical systems." We will explore deterministic chaos as well as regular (nonchaotic) dynamics. After an extended introduction to dynamical systems in general, including the Lorenz equations and the driven, damped pendulum, the course focuses on iterated maps of the interval. The main object of study is the logistic map family, which gives examples of the general theory of the sequence of period doubling bifurcations, the Schwarzian derivative, Misiurewicz points, Sharkovskii's theorem, and Feigenbaum's universal numbers.

Student Learning Outcomes: The student will gain an appreciation for the fact that simple systems can have complicated behavior. They will also learn that diverse nonlinear models within broad classes of systems have similar behavior. Two classes are dissipative systems, which have friction, and systems without friction for which energy is conserved.

Course Structure: Lecture format. Near the end of the semester, students will give presentations on their projects during class.

Course Outline:

Extended introduction to chaos and dynamical systems. Conservative and dissipative systems. Systems of Differential Equations, and Iterated maps. Examples of the logistic map, the Hénon map, and the Standard Map. Examples of the double pendulum, the van der Pol oscillator, the driven damped pendulum, and the Lorenz equations.

More mathematics about one-dimensional maps: fixed points and period k points, period three implies chaos, Sharkovskii's Theorem, the Schwarzian derivative, conjugacies and semi-conjugacies of maps, period doubling and Feigenbaum's constants.

Assessment of Student Learning Outcomes

The grade for the course will be determined by the following three components, which each having equal weight:

Homework: You know by now that it is necessary to practice math to learn it. You are *allowed* and *encouraged* to work together on homework. Some of the homework problems will require computer work. Some of the homework will involve computer work.

Project: You will do a research project, and make a 15 minute presentation to the class. The project can be done individually, or in a group of 2 or 3 people. Each person will give a presentation. This is not a Masters thesis, and it is OK if you are reproducing known results. However, it is not too hard in this relatively young field to find things that have not been done before. Our website has many suggestions for projects. The projects can be computer-based or not, depending on your inclination.

Exams: There will be one or two midterm exams and a comprehensive final exam.

Course Policies

University and Departmental Policies: Links to University and Departmental Policies are at the class website.

Amendments: Any changes to this syllabus will be announced in class, and an updated version will be posted at the class website. This version is dated January 20, 2023.