Sharkovskii order of $\mathbb{N}$, denoted $\prec$

$$1 \prec 2 \prec 4 \prec 8 \prec 16 \prec 32 \prec 64 \prec \cdots$$

$$\vdots$$

$$\cdots \prec 11\cdot 2^{n+1} \prec 9\cdot 2^{n+1} \prec 7\cdot 2^{n+1} \prec 5\cdot 2^{n+1} \prec 3\cdot 2^{n+1} \prec$$

$$\cdots \prec 11\cdot 2^{n} \prec 9\cdot 2^{n} \prec 7\cdot 2^{n} \prec 5\cdot 2^{n} \prec 3\cdot 2^{n} \prec$$

$$\vdots$$

$$\cdots \prec 11\cdot 4 \prec 9\cdot 4 \prec 7\cdot 4 \prec 5\cdot \prec 3\cdot 4 \prec$$

$$\cdots \prec 11\cdot 2 \prec 9\cdot 2 \prec 7\cdot 2 \prec 5\cdot 2 \prec 3\cdot 2 \prec$$

$$\cdots \prec 11 \prec 9 \prec 7 \prec 5 \prec 3$$

$\sup_{\prec} \{1,2,4,8,16,32,\ldots\}$ DNE

"Well-ordering" means every set has a smallest element

This shows that $\prec$ is not a well-ordering

$$\inf_{\prec} \{3,5,7,9,11,13,\ldots\} = 6$$