HW 1 problem 1, MAT 667, Dr. Jim

Body found at 3:00 am, 30°C
at 3:00 am, 29°C
at time of death, 37°C
Room temperature, 20°C

The next line, starting "Let" is super-important.
There are many choices, so be clear and specific.

Let $u(t)$ be the temperature (in °C) of the body at <u>t minutes after 3:00 am</u>

so $u(0)=30$, $u(60)=29$, $u(?)=37$ ← find this.

$\frac{du}{dt}=-k(u-20)$, $u(0)=30$ has the solution

Death occured about 302 minutes, or 5 hr. 2 min, befor 3:00 am.
Death occured about 9:58 pm

$$u(t)=20+10e^{-kt}$$

$$u(60)=20+10e^{-k\cdot 60}=29$$

$$10e^{-60k}=9$$

$$e^{-60k}=\frac{9}{10}, \text{ so } e^{-k}=\left(\frac{9}{10}\right)^{\frac{1}{60}}$$

$$\text{and } e^{-kt}=\left(\frac{9}{10}\right)^{\frac{t}{60}}$$

Note: 200 and 201 differ by 0.5%, but e^{20} and $e^{20.1}$ differ by 9.5%

e^{200} and e^{201} differ by 178%.

Thus $\boxed{u(t)=20+10\cdot\left(\frac{9}{10}\right)^{\frac{t}{60}}}$

<u>Note:</u> No rounding of logarithms.

Solve for t. Solution will be negative, giving time of death

$$u(t)=20+10\cdot\left(\frac{9}{10}\right)^{\frac{t}{60}}=37$$

$$10\cdot\left(\frac{9}{10}\right)^{\frac{t}{60}}=37-20=17, \quad \left(\frac{9}{10}\right)^{\frac{t}{60}}=\frac{17}{10}, \quad \frac{t}{60}\ln\left(\frac{9}{10}\right)=\ln\left(\frac{17}{10}\right)$$

$$t=60\,\frac{\ln(1.7)}{\ln(0.9)}\approx -302, \text{ using a calculator.}$$